CERN-TH/2000-285
SWAT/266
UGVA-DPT-00-9-1088
gr-qc/0012XXX

Accelerating Photons with Gravitational Radiation

G.M. Shore[1]

Département de Physique Théorique,
Université de Genève
24, quai E. Ansermet,
CH 1211 Geneva 4, Switzerland

and

TH Division, CERN,
CH 1211 Geneva 23, Switzerland

The nature of superluminal photon propagation in the gravitational field describing radiation from a time-dependent, isolated source (the Bondi-Sachs metric) is considered in an effective theory which includes interactions which violate the strong equivalence principle. Such interactions are, for example, generated by vacuum polarisation in conventional QED in curved spacetime. The relation of the resulting light-cone modifications to the Peeling Theorem for the Bondi-Sachs spacetime is explained.

CERN-TH/2000-285
SWAT/266
UGVA-DPT-00-9-1088
December 2000

[1] Permanent address: Department of Physics, University of Wales Swansea, Singleton Park, Swansea, SA2 8PP, U.K.

1. Introduction

The possibility of superluminal photon propagation in gravitational fields is an intriguing prediction of quantum field theory in curved spacetime. It raises fundamental questions about the realisation of causality and is potentially of great importance for early-universe cosmology.

In the context considered here, superluminal propagation was originally discussed in a pioneering paper by Drummond and Hathrell [1], who investigated the effects of vacuum polarisation on photon propagation in a variety of classical curved spacetimes, including Schwarzschild, Robertson-Walker and weak-field gravitational waves. They showed that in general it is possible to find directions and polarisations for which the photon velocity exceeds the fundamental constant c. This work has subsequently been generalised to other examples of background spacetimes, including Reissner-Nordstrom and Kerr black holes [2,3]. In ref.[4], we presented some further theoretical analysis, including the formulation of a polarisation sum rule and a horizon theorem, based on the original work of [1]. In this paper, we extend this development and present results for a phenomenologically important spacetime, the Bondi-Sachs metric describing gravitational radiation from an isolated source [5,6].

The essential physics underlying the Drummond-Hathrell mechanism for superluminal propagation is a violation of the strong equivalence principle. Here, we understand the weak equivalence principle (WEP) to be the requirement that spacetime is Riemannian and thus has at each point a local inertial frame (LIF). By the strong equivalence principle (SEP), we mean the further assumption that the laws of physics are the same in the LIFs at each point of spacetime, and take their special relativistic form at the origin of each LIF. In particular, this condition states that matter couples to the gravitational field only via the connection, with no direct curvature coupling. It is clear that the two forms of the equivalence principle have a quite different status. While the WEP is fundamental to the structure of general relativity, the SEP appears to be merely an extra dynamical assumption (minimal coupling) which may not be essential for the self-consistency of the theory. Of course, this is precisely what we are testing by studying superluminal propagation and causality in a situation where the SEP is relaxed. If the theory is indeed still self-consistent, it then becomes an experimental question whether or not SEP-violating interactions exist and with what strength. This paper is concerned with the characteristics of photon propagation in a particularly relevant gravitational field, that created by a time-dependent, isolated, radiating source.

The particular SEP-violating interactions we consider here are given by the effective action

$$\Gamma = \int dx\sqrt{-g}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{m^2}\Big(aRF_{\mu\nu}F^{\mu\nu} + bR_{\mu\nu}F^{\mu\lambda}F^{\nu}{}_{\lambda} + cR_{\mu\lambda\nu\rho}F^{\mu\lambda}F^{\nu\rho}\Big)\Big] \quad (1.1)$$

As shown by Drummond and Hathrell, this action arises through vacuum polarisation effects at one-loop level in QED in curved spacetime. In this case, $a = -\frac{1}{144}\frac{\alpha}{\pi}$, $b = \frac{13}{360}\frac{\alpha}{\pi}$ and $c = -\frac{1}{360}\frac{\alpha}{\pi}$ and the scale m is the electron mass. Further corrections to the effective action involving higher derivatives of the field strengths and curvatures also arise in QED and are relevant to the question of dispersion and high-frequency propagation, but these will not be considered here. Similar effective actions may also be expected to arise generically as low-energy approximations to more speculative fundamental theories of quantum gravity. However, at this point we do not need to restrict ourselves to any particular mechanism, but can instead consider the effective action (1.1) as a phenomenological model involving a new fundamental scale m to be determined by experiment. This is the approach we will adopt in this paper.

As reviewed in the next section, this action induces curvature-dependent modifications to the effective light cone governing photon propagation and leads to the possibility of superluminal velocities. Two important generic features were formalised in ref.[4]. First, for Ricci flat spacetimes there exists a polarisation sum rule whereby if one polarisation state propagates with velocity less than c, the other must necessarily have a velocity greater than c. Second, for black hole spacetimes, even if the photon velocity along some trajectory is different from c, it reverts to the standard light cone velocity precisely on the event horizon, ensuring that the physical and geometrical horizons coincide. These features are most apparent in the following formula for the effective light cone, following from eq.(1.1):

$$k^2 = -\frac{16\pi}{m^2}(b+2c)T_{\mu\nu}k^{\mu}k^{\nu} + \frac{8c}{m^2}C_{\mu\lambda\nu\rho}k^{\mu}k^{\nu}a^{\lambda}a^{\rho} \quad (1.2)$$

where k^{μ} is the wave vector (photon momentum) and a^{λ} is the polarisation. In this form, we have used the Einstein equations (with $G = 1$) to substitute the energy-momentum tensor $T_{\mu\nu}$ for the Ricci tensor. The first term is polarisation-independent and is proportional to the projection of the energy-momentum tensor appearing in the weak energy condition, viz. $T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ for any null vector k^{μ}. A similar modification to k^2 also arises for photon propagation in other modified environments such as background electromagnetic fields or finite temperature. The second contribution, which is special to gravitational backgrounds, produces a polarisation dependent shift in the effective light cone proportional to a particular projection of the Weyl tensor (i.e. a particular Newman-Penrose scalar, as explained in section 3). The shift is equal and opposite for the two polarisations, one of which is therefore necessarily superluminal.

A polarisation-dependent shift in the light cone (gravitational birefringence) allowing superluminal propagation is therefore the essential signature of SEP-violating interactions in curved spacetime electrodynamics.

If this effect is indeed observable, there are a number of potentially important cases which should be considered. The first, already discussed in ref.[1], is the polarisation dependence of the bending of light by a star or other matter distribution whose exterior field is described by the Schwarzschild metric. In this case, the angle of deflection becomes dependent on the photon polarisation. Superluminal propagation would be especially important in the evolution of the early universe, where it could be relevant for the horizon problem [7,8,9,10]. This probably requires changes in c by many orders of magnitude, rather than the perturbative corrections considered here (we are implicitly considering (1.1) to be the leading terms in an expansion in $O(R/m^2)$). Nevertheless, (1.2) does predict superluminal propagation for FRW spacetimes [1]. Here, the spatial isotropy ensures that the second term vanishes ($C_{\mu\lambda\nu\rho} = 0$ for FRW spacetime), but the first term produces a polarisation-independent shift in the light cone which implies superluminal velocities provided the weak energy condition $\rho + p \geq 0$ is satisfied. On the other hand, for de Sitter spacetime, k^2 remains zero because the spacetime is isotropic.

Here, we consider a third important case: the propagation of light in the background spacetime corresponding to a time-dependent, isolated source emitting gravitational radiation. We show that there is a polarisation-dependent superluminal effect and express this in terms of the appropriate Newman-Penrose scalars characterising the Weyl tensor. The asymptotic behaviour depends on the direction of the photons. For ingoing photons, the light cone shift is of $O(1/r)$ and is given by the amplitude of the gravitational waves far from the source; for the more important case of outgoing photons, the shift is $O(1/r^5)$ and depends on the 'quadrupole aspect' of the Bondi-Sachs metric. These results are related to the Peeling Theorem [6], which describes the asymptotic dependence of the Newman-Penrose scalars in the Bondi-Sachs metric.

Before deriving these results, we mention briefly two interesting recent developments in the theory of superluminal propagation. First, Albrecht, Barrow and Magueijo [8,9,10] have studied the implications for the early universe of allowing the fundamental constant c to be time-dependent. Although this formalism is very different from that considered here, these papers illustrate well the potential importance of superluminal effects in cosmology. Second, Drummond [11] has recently proposed a manifestly covariant bi-metric theory of gravity in which matter and gravity couple to different vierbeins whose relative orientation is determined dynamically through a sigma model action. He then applies this theory to the specific problem of 'dark matter', arguing that the phenomena usually attributed to the existence of dark matter can alternatively be described by a modification of the fundamental theory of gravity. This is of course a much more radical generalisation of conventional gravity than considered in this paper, but this new theory would certainly incorporate the type of effects discussed here.

In the rest of the paper, we review briefly the derivation of the modified light cone condition (1.2) in section 2. Then, in section 3, we describe the Bondi-Sachs metric and the Peeling Theorem. The results on photon propagation and our general conclusions are given in sections 4 and 5.

2. Superluminal Propagation in Gravitational Fields

The characteristics of photon propagation following from the effective action (1.1) are most easily described using geometric optics [1]. In this picture, the electromagnetic field is written as the product of a slowly-varying amplitude and a rapidly-varying phase, i.e.

$$F_{\mu\nu} = f_{\mu\nu} e^{i\vartheta} \tag{2.1}$$

The wave vector (photon momentum) is defined as $k_\mu = \partial_\mu \vartheta$, while the Bianchi identity constrains $f_{\mu\nu}$ to have the form

$$f_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu \tag{2.2}$$

where the direction of a^μ specifies the polarisation. For physical polarisations, $k_\mu a^\mu = 0$.

For conventional curved spacetime QED based on the usual Maxwell action, the equation of motion is simply

$$D_\mu F^{\mu\nu} = 0 \qquad \Rightarrow \qquad k_\mu f^{\mu\nu} = 0 \tag{2.3}$$

Since this implies

$$k^2 a^\nu = 0 \tag{2.4}$$

we immediately deduce that $k^2 = 0$, i.e. k^μ is a null vector. Then, from its definition as a gradient, we see

$$k^\mu D_\mu k^\nu \;=\; k^\mu D^\nu k_\mu \;=\; \frac{1}{2} D^\nu k^2 \;=\; 0 \tag{2.5}$$

Light rays (photon trajectories) are defined as the integral curves of the wave vector, i.e. the curves $x^\mu(s)$ where $\frac{dx^\mu}{ds} = k^\mu$. These curves therefore satisfy

$$0 \;=\; k^\mu D_\mu k^\nu \;=\; \frac{d^2 x^\nu}{ds^2} + \Gamma^\nu_{\mu\lambda} \frac{dx^\mu}{ds}\frac{dx^\lambda}{ds} \tag{2.6}$$

which is the geodesic equation. So in the conventional theory, light rays are null geodesics.

The effective action (1.1) gives rise to a modified equation of motion which, under the approximations listed below, implies

$$k_\mu f^{\mu\nu} \;+\; \frac{1}{m^2}\Big[2bR^\mu{}_\lambda k_\mu f^{\lambda\nu} + 4cR^{\mu\nu}{}_{\lambda\rho}k_\mu f^{\lambda\rho}\Big] \;=\; 0 \tag{2.7}$$

Here, we have made the standard geometric optics approximation of neglecting derivatives of $f_{\mu\nu}$ relative to the derivatives of the phase factor, which produce powers of the momentum k_μ; we have neglected derivatives of the curvature tensor, which would produce corrections of $O(\lambda/L)$, where λ is the photon wavelength and L is a typical curvature scale; and we have omitted the corrections to the term $D_\mu F^{\mu\nu}$ of $O(aR/m^2)$ coming from the Ricci scalar in the effective action – these are of $O(\lambda_c^2/L^2)$, where λ_c is the Compton wavelength corresponding to a particle of mass m (the electron in the conventional QED derivation of (1.1)) which should be neglected as higher order if we view (1.1) as the leading term in an expansion in powers of curvature.

Eq.(2.7) can now be rewritten as an equation for the polarisation vector a^μ, and re-expressing in terms of the Weyl tensor we find

$$k^2 a^\nu \;+\; \frac{(2b+4c)}{m^2}\; R_{\mu\lambda}\big(k^\mu k^\lambda a^\nu - k^\mu k^\nu a^\lambda\big) \;+\; \frac{8c}{m^2}\; C_\mu{}^\nu{}_{\lambda\rho}k^\mu k^\lambda a^\rho \;=\; 0 \tag{2.8}$$

The solutions of this equation describe the characteristics of propagation for a photon of momentum k^μ and polarisation a^μ. Contracting with a^μ (and assuming spacelike normalisation $a^\mu a_\mu = -1$), we find the effective light cone

$$k^2 \;+\; \frac{(2b+4c)}{m^2}R_{\mu\lambda}k^\mu k^\lambda \;-\; \frac{8c}{m^2}C_{\mu\nu\lambda\rho}k^\mu k^\lambda a^\nu a^\rho \;=\; 0 \tag{2.9}$$

from which (1.2) follows immediately.

It should be noted that all these equations are manifestly local Lorentz invariant. On the other hand, the presence of the explicit curvature coupling in the effective action means that the equations of motion do not reduce to their special relativistic form at the origin of each LIF, and thus that the dynamics is different in the LIFs at different points in spacetime. In this sense, these equations violate the strong principle of equivalence. Some implications of this for causality have been discussed in refs.[1,4,12].

At this point, it is illuminating to re-write the effective light cone condition using the Newman-Penrose tetrad formalism, and in particular to show the dependence on the NP scalars characterising the Weyl tensor. The first step is to choose a null tetrad as follows. Let ℓ^μ be a null vector. Let a^μ and b^μ be spacelike, transverse vectors and define the complex null vectors m^μ and $\bar{m}^\mu$ by $m^\mu = \frac{1}{\sqrt{2}}(a^\mu + ib^\mu)$ and $\bar{m}^\mu = \frac{1}{\sqrt{2}}(a^\mu - ib^\mu)$. Finally, choose a further null vector n^μ orthogonal to m^μ and $\bar{m}^\mu$. These vectors satisfy the orthogonality conditions:

$$\ell.m \;=\; \ell.\bar{m} \;=\; n.m \;=\; n.\bar{m} \;=\; 0 \tag{2.10}$$

they are null vectors:

$$\ell.\ell \;=\; n.n \;=\; m.m \;=\; \bar{m}.\bar{m} \;=\; 0 \tag{2.11}$$

and we impose the normalisation conditions:

$$\ell.n \;=\; 1 \qquad\qquad\qquad m.\bar{m} \;=\; -1 \tag{2.12}$$

The null tetrad is defined by the vierbeins e_a^μ, where we define $e_1^\mu = \ell^\mu$, $e_2^\mu = n^\mu$, $e_3^\mu = m^\mu$ and $e_4^\mu = \bar{m}^\mu$. The corresponding metric is

$$\eta_{ab} \;=\; \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{2.13}$$

The five complex NP scalars characterising the Weyl tensor are (following closely the notation of ref.[13])

$$\begin{aligned}
\Psi_0 &= -C_{abcd}\ell^a m^b \ell^c m^d &&= -C_{1313} \\
\Psi_1 &= -C_{abcd}\ell^a n^b \ell^c m^d &&= -C_{1213} \\
\Psi_2 &= -C_{abcd}\ell^a m^b \bar{m}^c n^d &&= -C_{1342} \\
\Psi_3 &= -C_{abcd}\ell^a n^b \bar{m}^c n^d &&= -C_{1242} \\
\Psi_4 &= -C_{abcd}n^a \bar{m}^b n^c \bar{m}^d &&= -C_{2424}
\end{aligned} \tag{2.14}$$

The symmetries of the Weyl tensor imply several interesting relations amongst its components. Most important for our discussion are the trace-free conditions

$$\eta^{ad} C_{abcd} = 0 \tag{2.15}$$

and cyclicity, e.g.

$$C_{1234} + C_{1342} + C_{1423} = 0 \tag{2.16}$$

Together, these imply the important identity

$$C_{\mu\nu\lambda\rho}\ell^\mu m^\nu \ell^\lambda \bar{m}^\rho \;=\; C_{1314} \;=\; 0 \tag{2.17}$$

Components of the Ricci tensor have a similar classification. We define

$$\begin{aligned}
\Phi_{00} &= -\frac{1}{2}R_{\mu\nu}\ell^\mu\ell^\nu = -\frac{1}{2}R_{11} & \Phi_{22} &= -\tfrac{1}{2}R_{\mu\nu}n^\mu n^\nu = -\tfrac{1}{2}R_{22} \\
\Phi_{02} &= -\frac{1}{2}R_{\mu\nu}m^\mu m^\nu = -\frac{1}{2}R_{33} & \Phi_{20} &= -\tfrac{1}{2}R_{\mu\nu}\bar{m}^\mu\bar{m}^\nu = -\tfrac{1}{2}R_{44} \\
\Phi_{01} &= -\frac{1}{2}R_{\mu\nu}\ell^\mu m^\nu = -\frac{1}{2}R_{13} & \Phi_{10} &= -\tfrac{1}{2}R_{\mu\nu}\ell^\mu\bar{m}^\nu = -\tfrac{1}{2}R_{14} \\
\Phi_{12} &= -\frac{1}{2}R_{\mu\nu}n^\mu m^\nu = -\frac{1}{2}R_{23} & \Phi_{21} &= -\tfrac{1}{2}R_{\mu\nu}n^\mu\bar{m}^\nu = -\tfrac{1}{2}R_{24}
\end{aligned}$$

$$\begin{aligned}
\Phi_{11} &= -\frac{1}{4}(R_{\mu\nu}\ell^\mu n^\nu + R_{\mu\nu}m^\mu\bar{m}^\nu) = -\frac{1}{4}(R_{12}+R_{34}) \\
\Lambda &= \frac{1}{24}R = \frac{1}{12}(R_{\mu\nu}\ell^\mu n^\nu - R_{\mu\nu}m^\mu\bar{m}^\nu) = \frac{1}{12}(R_{12}-R_{34})
\end{aligned} \tag{2.18}$$

We can now re-express the light cone condition in NP form, using the identity (2.17). For example, if we choose the (unperturbed) photon momentum in the direction of the null vector ℓ^μ, i.e. $k^\mu = \omega\ell^\mu$, and the transverse polarisation vectors as a^μ, b^μ, then the term in (2.9) proportional to the Weyl tensor becomes, for the two polarisations respectively,

$$\begin{aligned}
\pm\frac{1}{2}C_{\mu\nu\lambda\rho}\ell^\mu\ell^\lambda\big(m^\nu \pm \bar{m}^\nu\big)\big(m^\rho \pm \bar{m}^\rho\big) &= \pm\frac{1}{2}\big(C_{\mu\nu\lambda\rho}\ell^\mu m^\nu\ell^\lambda m^\rho + C_{\mu\nu\lambda\rho}\ell^\mu\bar{m}^\nu\ell^\lambda\bar{m}^\rho\big) \\
&= \pm\frac{1}{2}(\Psi_0 + \Psi_0^*)
\end{aligned} \tag{2.19}$$

The lightcone condition is therefore simply

$$k^2 = \frac{(4b+8c)\omega^2}{m^2}\Phi_{00} \pm \frac{4c\omega^2}{m^2}(\Psi_0 + \Psi_0^*) \tag{2.20}$$

depending on the polarisation. It is interesting that this depends on only a single NP scalar for each of the Ricci and Weyl tensor contributions. The polarisation sum rule and horizon theorem described in ref.[4] are immediate consequences of this form of the light-cone condition. Other choices of the photon momentum and polarisation give analogous expressions for the modified light cone.

In the rest of this paper, we apply this result to the special example of the Bondi-Sachs metric and show the precise relation to the Peeling Theorem.

3. Bondi-Sachs Metric and the Peeling Theorem

The spacetime describing an isolated, radiating source is given by the Bondi-Sachs metric:

$$ds^2 = -W du^2 - 2e^{2\beta} du dr + r^2 h_{ij}(dx^i - U^i du)(dx^j - U^j du) \tag{3.1}$$

where

$$h_{ij}dx^i dx^j = \frac{1}{2}(e^{2\gamma} + e^{2\delta})d\theta^2 + 2\sinh(\gamma - \delta)\sin\theta d\theta d\phi + \frac{1}{2}(e^{-2\gamma} + e^{-2\delta})\sin^2\theta d\phi^2 \tag{3.2}$$

The metric is valid in the vicinity of future null infinity $\mathcal{I}^+$. The family of hypersurfaces $u = const$ are null, i.e. $g^{\mu\nu}\partial_\mu u \partial_\nu u = 0$. Their normal vector ℓ_μ satisfies

$$\ell_\mu = \partial_\mu u \qquad \Rightarrow \qquad \ell^2 = 0, \qquad \ell^\mu D_\mu \ell^\nu = 0 \tag{3.3}$$

The curves with tangent vector ℓ^μ are therefore null geodesics; the coordinate r is a radial parameter along these rays and is identified as the luminosity distance.

The six independent functions characterising the metric have the following asymptotic expansions near $\mathcal{I}^+$ for large r:

$$\begin{aligned}
W &= 1 - \frac{2\mathcal{M}}{r} + O(\frac{1}{r^2}) \\
\beta &= -\frac{1}{4}(c_+^2 + c_\times^2)\frac{1}{r^2} + O(\frac{1}{r^3}) \\
\frac{1}{2}(\gamma + \delta) &= \frac{c_+}{r} + \frac{q_+}{r^3} + O(\frac{1}{r^4}) \\
\frac{1}{2}(\gamma - \delta) &= \frac{c_\times}{r} + \frac{q_\times}{r^3} + O(\frac{1}{r^4}) \\
U^\theta + i\sin\theta U^\phi &= -\frac{1}{\sin^2\theta}\Big(\partial_\theta - \frac{i}{\sin\theta}\partial_\phi\Big)\Big(\sin^2\theta(c_+ + ic_\times)\Big)\frac{1}{r^2} \\
&\qquad + 2\Big(d^\theta + i\sin\theta d^\phi + \ldots\Big)\frac{1}{r^3} + O(\frac{1}{r^4})
\end{aligned} \tag{3.4}$$

where $\mathcal{M}$, $c_{+(\times)}$, $q_{+(\times)}$, $d^{\theta(\phi)}$ are all functions of (u, θ, ϕ). The form of these expansions follows from a careful analysis of the characteristic initial-value problem for the vacuum Einstein equations (see refs.[5,6] and for a textbook review [14]).

The function $\mathcal{M}(u, \theta, \phi)$ may be called [14] the 'mass aspect'. Its integral over the unit 2-sphere,

$$M(u) = \frac{1}{4\pi}\int d\Omega_2 \ \mathcal{M}(u, \theta, \phi) \tag{3.5}$$

represents the mass of the system at $\mathcal{I}^+$ and is the familiar Bondi mass. Similarly $d^{\theta(\phi)}$ and $q_{+(\times)}$ are the 'dipole' and 'quadrupole aspects' respectively. $\mathcal{M}$, $d^{\theta(\phi)}$ and $q_{+(\times)}$ satisfy dynamical equations derived from the Einstein equations for their $u-$derivatives, involving also the remaining functions $c_{+(\times)}$.

$\partial_u c_{+(\times)}$ are functions which must be given as initial data and are specified on $\mathcal{I}^+$. They are the Bondi 'news functions'. An especially important result relates $\partial_u \mathcal{M}(u)$ to the news functions:

$$\partial_u \mathcal{M}(u) = -\frac{1}{4\pi} \int d\Omega_2 \, \Big((\partial_u c_+)^2 + (\partial_u c_\times)^2 \Big) \tag{3.6}$$

This states that the Bondi mass is reduced if the news function is non-zero, corresponding to the fact that the system loses mass if and only if it is radiating.

Finally, as we discuss later, the second derivatives $\partial_u^2 c_{+(\times)}$ can be identified as the amplitude of the gravitational waves in a weak-field limit. Notice also that $\gamma \pm \delta$, and hence $c_{+(\times)}$ and $q_{+(\times)}$, correspond to the two independent gravitational wave polarisations.

We return to the interpretation of $c_{+(\times)}$ in section 4 after introducing the Peeling Theorem, discussed by Sachs in [6]. This gives the leading asymptotic behaviour in $1/r$ of the set of Newman-Penrose scalars $\Psi_0, \ldots, \Psi_4$ characterising the Weyl tensor in the Bondi-Sachs spacetime. Although we do not need to exploit it here, the $\Psi_0, \ldots, \Psi_4$ are intimately related to the Petrov classification, which classifies the Weyl tensor according to the degeneracy of its principal null vectors.

For our purposes, we simply need the following result (note that since we only need the precise results for Ψ_0 and Ψ_4 in what follows, we have only written schematic forms for the others):

$$\begin{aligned}
\Psi_4 &= \frac{1}{r} \Big[\partial_u^2 (c_+ - i c_\times) \Big] \\
\Psi_3 &\sim \frac{1}{r^2} \Big[\frac{1}{\sin^2\theta} \Big(\partial_\theta + \frac{i}{\sin\theta} \partial_\phi \Big) \Big(\sin^2\theta \, \partial_u (c_+ - i c_\times) \Big) \Big] \\
\Psi_2 &\sim \frac{1}{r^3} \Big[\mathcal{M} + \Big(\partial_\theta + \frac{i}{\sin\theta} \partial_\phi \Big) \Big(c_+^2 + c_\times^2 \Big) \Big] \\
\Psi_1 &\sim \frac{1}{r^4} \Big[d^\theta + i \sin\theta \, d^\phi \Big] \\
\Psi_0 &= \frac{1}{r^5} \Big[6(q_+ - i q_\times) \Big]
\end{aligned} \tag{3.7}$$

where we have set $\ell_\mu = \partial_\mu u$ and chosen the transverse vector $m_\mu = \frac{1}{\sqrt{2}} r(\partial_\mu \theta + i \sin\theta \partial_\mu \phi)$.

The essence of the Peeling Theorem is the correlation between the leading order in $1/r$ and the type of the NP scalar. Notice also that the leading coefficients as we pass from Ψ_4 to Ψ_0 involve respectively $\partial_u^2 c$, $\partial_u c$, $\mathcal{M}$, d and q, with the higher moment aspects being associated with successively higher powers of $1/r$.

4. Photon Propagation in the Bondi-Sachs Spacetime

We are now ready to combine the general results for modified photon propagation in section 2 with the special features of the Bondi-Sachs gravitational radiation spacetime. Consider first the case of photons with momentum $k^\mu = \omega \ell^\mu$. This corresponds to motion radially outwards from the gravitationally radiating source. (To confirm this, note that the equipotential surfaces for outgoing waves are $\vartheta(u) = const$, so using the geometrical optics analysis above, the corresponding rays have tangent vector $k_\mu = \partial_\mu \vartheta$, so we can identify the directions of k_μ and ℓ_μ.) Choose the transverse polarisation vectors a^μ, b^μ to lie in the θ and ϕ directions respectively, so that

$$
\begin{aligned}
a^\mu &= \frac{1}{\sqrt{2}}(m^\mu + \bar{m}^\mu) \\
b^\mu &= -\frac{i}{\sqrt{2}}(m^\mu - \bar{m}^\mu)
\end{aligned}
\tag{4.1}
$$

with m^μ as in section 3. In this case, according to eq.(2.20) the light-cone shift for the two polarisations is

$$
k^2 \quad = \quad \pm \frac{4c\omega^2}{m^2} \left(\Psi_0 + \Psi_0^*\right) \tag{4.2}
$$

which for the Bondi-Sachs metric is

$$
k^2 \quad = \quad \pm \frac{48c\omega^2}{m^2} \frac{q_+}{r^5} \tag{4.3}
$$

corresponding to a velocity shift δv proportional to $\pm q_+/r^5$. As usual for Ricci-flat spacetimes, the two transverse polarisations have equal and opposite velocity shifts, so one is always superluminal. For outgoing photons, therefore, we do find a velocity shift, but it is very weak, falling off as $1/r^5$, and is governed by the quadrupole aspect q_+ of the gravitational field.

It is interesting also to look at the photons with polarisation vectors rotated through 45^o. In this case, we choose

$$
\begin{aligned}
a^\mu &= \frac{1}{2}(m^\mu + \bar{m}^\mu) - \frac{i}{2}(m^\mu - \bar{m}^\mu) \\
b^\mu &= -\frac{1}{2}(m^\mu + \bar{m}^\mu) - \frac{i}{2}(m^\mu - \bar{m}^\mu)
\end{aligned}
\tag{4.4}
$$

A calculation along the lines of eq.(2.19) now gives

$$
k^2 \quad = \quad \pm \frac{4c\omega^2}{m^2} \, i\left(\Psi_0 - \Psi_0^*\right) \tag{4.5}
$$

For the Bondi-Sachs metric this is

$$k^2 \;=\; \pm\frac{48c\omega^2}{m^2}\;\frac{q_\times}{r^5} \tag{4.6}$$

As expected, the photons with polarisations rotated through 45^o are influenced by the $\times$ polarisation of the gravitational radiation, compared with the original choice aligned with the $+$ polarisation.

A significantly larger effect is obtained if we consider incoming photons, moving radially towards the source of gravitational radiation. In this case, we have $k^\mu = \omega n^\mu$. For the initial choice of photons polarised in the θ or ϕ directions, we find

$$k^2 \;=\; \pm\frac{4c\omega^2}{m^2}\;\big(\Psi_4 + \Psi_4^*\big) \tag{4.7}$$

while for the polarisations rotated through 45^o we have

$$k^2 \;=\; \pm\frac{4c\omega^2}{m^2}\;i\big(\Psi_4 - \Psi_4^*\big) \tag{4.8}$$

In Bondi-Sachs, this means

$$k^2 \;=\; \pm\frac{8c\omega^2}{m^2}\;\frac{1}{r}\partial_u^2 c_+ \tag{4.9}$$

and

$$k^2 \;=\; \pm\frac{8c\omega^2}{m^2}\;\frac{1}{r}\partial_u^2 c_\times \tag{4.10}$$

respectively. In this case, the superluminal velocity shifts are of $O(1/r)$, i.e. δv is proportional to $\partial_u^2 c_{+(\times)}/r$ depending on whether the photon polarisation is aligned with the $+$ or $\times$ gravitational radiation polarisation.

We now see clearly the relation of superluminal photon propagation to the Peeling Theorem. Depending on the direction (and polarisation) of the photons, the shift in the light cone is proportional to one of the NP scalars characterising the Weyl tensor. The Peeling Theorem specifies the leading order in $1/r$ of each of these types in the vicinity of $\mathcal{I}^+$, which translates immediately into a result giving the $1/r$-dependence of the photon velocity shifts.

These results of course hold for the full gravitational radiation field described by the Bondi-Sachs metric. It is interesting at this point to compare them with those obtained previously for weak-field gravitational radiation in the linearised approximation. To see this relation, consider the following metric describing gravitational plane waves in the linearised, weak-field limit where the metric perturbation $h_{\mu\nu}$ from Minkowski spacetime is chosen in transverse, traceless gauge:

$$ds^2 = -dudv + \big(1 - h_+(u)\big)dx^2 + \big(1 + h_+(u)\big)dy^2 - 2h_\times(u)dxdy \tag{4.11}$$

h_+ and $h_\times$ of course correspond to the $+$ and $\times$ polarised gravitational waves, and $\partial_u^2 h_{+(\times)}$ represent their amplitudes [14]. The relevant components of the Weyl tensor are:

$$C_{uxux} = -C_{uyuy} = -\frac{1}{2}\partial_u^2 h_+$$
$$C_{uxuy} = -\frac{1}{2}\partial_u^2 h_\times \qquad (4.12)$$

Linearising the Bondi-Sachs metric in γ and δ and discarding the functions W, β and U^i, we find

$$ds^2 = -dudv + r^2\Big((1+\gamma+\delta)d\theta^2 + 2(\gamma-\delta)\sin\theta d\theta d\phi + (1-\gamma-\delta)\sin^2\theta d\phi^2\Big) \qquad (4.13)$$

which has therefore reduced to the weak-field gravitational wave metric with

$$h_+ = -(\gamma+\delta) = -2\frac{c_+}{r} \qquad\qquad h_\times = -(\gamma-\delta) = -2\frac{c_\times}{r} \qquad (4.14)$$

confirming the identification already assumed above that $c_{+(\times)}$ correspond to the two independent gravitational wave polarisations.

Returning to the metric (4.11), the light-cone shift for photons travelling in the opposite direction to the gravitational waves is

$$k^2 = \pm\frac{8c\omega^2}{m^2}C_{uxux} = \pm\frac{4c\omega^2}{m^2}\partial_u^2 h_+ \qquad (4.15)$$

for x, y polarised photons, and

$$k^2 = \pm\frac{8c\omega^2}{m^2}C_{uxuy} = \pm\frac{4c\omega^2}{m^2}\partial_u^2 h_\times \qquad (4.16)$$

for 45^o rotated photons. With the weak-field identifications (4.14), we recover (4.9), (4.10).

On the other hand, for photons travelling in the same direction as the gravitational waves, the light-cone shifts are proportional to the C_{vxvx}, C_{vyvy} and C_{vxvy} components of the Weyl tensor, which vanish. For both polarisations, therefore, $k^2 = 0$.

To summarise, for weak-field gravitational plane waves, the effect on photon velocity is as follows. Photons travelling in the same direction as the gravitational waves feel no effect, whereas photons travelling in the opposite direction experience velocity shifts proportional to the gravitational wave amplitudes $\partial_u^2 h_{+(\times)}$, depending on the alignment of the photon and gravitational wave polarisations.

In the full Bondi-Sachs radiation metric, a similar situation holds, except that here the photons travelling in the same direction as the gravitational radiation also experience a velocity shift proportional to $q_{+(\times)}$, though only of $O(1/r^5)$, while those travelling in the opposite direction experience a much larger $O(1/r)$ effect proportional to $\partial_u^2 c_{+(\times)}$.

5. Conclusion

If the strong equivalence principle is violated, photons do not necessarily propagate along the null geodesics of the background curved spacetime. The physical light cone is shifted with respect to the geometrical one. Here, we have discussed this effect in terms of an effective field theory containing explicit SEP-violating interactions. Such terms have been shown to arise even in conventional QED in curved spacetime through vacuum polarisation effects and are expected to appear generically in the low-energy limit of theories of quantum gravity.

Developing earlier work on black hole spacetimes, we have considered the special case of the gravitational radiation metric introduced by Bondi, van der Burg and Metzner [5] and by Sachs [6]. This has allowed us to generalise previous results on photon propagation in weak-field gravitational wave backgrounds obtained by Drummond and Hathrell [1].

Our principal results are that the velocity shifts are either superluminal or subluminal depending on which of the transverse photon polarisations is considered. For photons travelling inward towards the source of gravitational radiation, the velocity shifts are asymptotically of $O(1/r)$ and depend on the functions $\partial_u^2 c_{+(\times)}$, where $\partial_u c_{+(\times)}$ are the Bondi news functions. The light-cone shift depends on the relative alignment of the photon and gravitational radiation polarisations. For photons moving outwards along the direction of the gravitational radiation, we still find a non-vanishing velocity shift (in contrast to the case of weak-field gravitational waves) controlled by the quadrupole aspect $q_{+(\times)}$ of the gravitational radiation, but it is only of $O(1/r^5)$.

The implications of these results for the question of causality in the presence of superluminal propagation are left for future work.

6. Acknowledgments

I am grateful to I. Drummond, R. Durrer, K. Kunze, M. Maggiore, W. Perkins and G. Veneziano for helpful and interesting discussions. This research is supported in part by PPARC grant GR/L56374.

References

[1] I.T. Drummond and S. Hathrell, Phys. Rev. D22 (1980) 343

[2] R.D. Daniels and G.M. Shore, Nucl. Phys. B425 (1994) 634

[3] R.D. Daniels and G.M. Shore, Phys. Lett. B367 (1996) 75

[4] G.M. Shore, Nucl. Phys. B460 (1996) 379

[5] H. Bondi, M.G.J. van der Burg and A.W.K. Metzner, Proc. Roy. Soc. A269 (1962) 21

[6] R.K. Sachs, Proc. Roy. Soc. A270 (1962) 103

[7] I.T. Drummond, private communication

[8] J.D. Barrow and J. Magueijo, Class. Quant. Grav. 16 (1999) 1435

[9] A. Albrecht and J. Magueijo, Phys. Rev. D59 (1999) 043516

[10] J. Magueijo, Phys. Rev. D62 (2000) 103521

[11] I.T. Drummond, gr-qc/9908058

[12] A.D. Dolgov and I.D. Novikov, Phys. Lett. B442 (1998) 82

[13] S. Chandresekhar, *The Mathematical Theory of Black Holes*, Clarendon, Oxford (1985)

[14] R.A. d'Inverno, *Introducing Einstein's Relativity*, Clarendon, Oxford (1992)